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Anthropogenic and Natural Forcings as Functions of Emission Time

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Abstract

Accurate projection of the climate parameters as a result of anthropogenic emissions has merit at the societal and scientific levels. This, however, has had a high degree of uncertainty because aspects of climate change are missing in the current science. Emission time is an argument of the forcing functions of the climate agents and the science does not account for this fundamental independent variable. Life on earth is the other function variable and natural variability should therefore be discarded for long-term climate projection. These findings are demonstrated mathematically in this manuscript. Accordingly, simple yet accurate mathematical equations are derived to calculate anthropogenic and natural forcings. They serve as the basis for accurate projection of sea level, surface temperature, relative humidity, and other climate parameters with greenhouse gas emissions. The equations are validated based on the present findings of the Intergovernmental Panel on Climate Change, past climates, and the work of others and are found to be in agreement.

Keywords

Anthropogenic Forcing; Natural Forcing; Volcanic Aerosols; Greenhouse Gases; Emission Time

Introduction

Reference [1] summarizes the current understanding of the climate science, which is generally based on the assessment reports of the Intergovernmental Panel on Climate Change (IPCC). These reports are summaries derived from a large number of publications prepared by researchers around the world. Dominant climate drivers are concentrations of greenhouse gases, in particular carbon dioxide. Greenhouse gas emissions are thought to trap heat radiated from the warmer surface that otherwise would be radiated to space. The earth and surface heat budgets increase as a result. This physical explanation, known as the greenhouse gas effect, is not completely understood and has not been fully endorsed by the public, example [2]. The gases released into the atmosphere have the atmospheric air temperature; they cannot transfer heat to the warmer surface without the expenditure of external mechanical energy. Such energy is unavailable to these gases. Based on the laws of thermodynamics, the physical explanation is questionable. The work presented in this manuscript reveals a different physical explanation based on valid equations of physics and chemistry that have passed the test of time. Because carbon dioxide is the prevailing greenhouse gas and its molecular weight is greater than that of air, the average molecular weight of the air mixture increases with carbon dioxide emissions. The lapse rate must decrease further and the temperature profile of the atmospheric air must decrease as well. The energy radiated to space is reduced, which is observed, [3], and the surface accumulates heat. The effect of the observed small quantities of carbon dioxide emitted into the atmospheric air is not negligible because it is amplified by the sheer size of atmosphere and time.

In addition, emission time is an independent variable of carbon dioxide forcing; it is an argument of the mathematical forcing function and the existing science does not account for this fact. Past and present climate data unquestionably reveal that carbon dioxide forcing is time dependent. From Antarctic Ice Core Data presented in [4], carbon dioxide concentration in the atmosphere increased by about 55 parts per million by volume, ppmv, during the warming campaign between 18 000 Before Present (BP) and 13 750 BP. Simultaneously, the average surface temperature increased by 4 ^{0}K to 6 ^{0}K. For the present warming trend, approximately 260 years, carbon dioxide concentration has increased by slightly over 100 ppmv, yet the observed increase of the average surface temperature is less than 1 ^{0}K. Clearly, carbon dioxide forcing is time dependent, which was not accounted for in the forcing equations used by the early IPCC assessment reports, [5, 6]. The equations are mathematical relationships between forcing of carbon dioxide and its concentration in the atmosphere. Emission time is not an argument of the forcing equations. They are presented and discussed in published literature such as [7, 8, 9] that can be referred to for further reading on this subject. From mathematical perspectives, the concentration of carbon dioxide, which is time dependent, is the only argument of the forcing function based on the current climate understanding. Or, $N=f[C(t)]$. Where N is carbon dioxide forcing function, C is the concentration of carbon dioxide in the atmosphere, and, t, is emission time. In this case, the change of forcing with time $dN/dt=(df/dC) \times (dC/dt)$, and forcing infinitesimal change $dN=(df/dC) \times (dC/dt) \times dt$. The work presented in this manuscript reveals a different forcing function for carbon dioxide of this nature $N=f[t, C]$. Or emission time and concentration are arguments of the forcing function. Therefore, forcing variation with time $dN/dt=\partial f/\partial t$, and forcing infinitesimal change $dN=\partial f/\partial t \times dt+\partial f/\partial C \times dC$, which is considerably different from what is currently used in the climate science.

Presently, the IPCC utilizes climate computer models for their assessment of present and future trends of surface temperature, sea level, and other climate parameters. Chapter 9 of [10] is dedicated to the evaluation of climate models. While the models produce the general feature of climate change, there is room for improvement. Climate projection is a long-term assessment of climate parameters where natural variability has virtually no contribution based on [10], Chapter 8. A similar conclusion is reached by [11] regarding the observed sea level rise. The trend of sea level rise is independent of El Niño Southern Oscillations (ENSO). Accordingly, short-term natural variability can be neglected for long-term climate projection and the effect of greenhouse gas emissions only should be considered.

Past climates were driven by variations of the concentration of carbon dioxide in the atmosphere based on Antarctic Ice Core Data, [4]. The natural variability of past climates is unnoticed in the temperature profile. Their negligible impact is also evident in sea level and surface temperature data available to us relative to the current warming trend. This holds true for ENSO and volcanic eruptions as the record shows, [11, 12, 13] to name a few. Reference [10] assigns near-zero value to natural forcing. The record clearly shows that at the conclusion of the cycle of these natural events, the observed surface and air temperatures return back to their original values prior to the cycle. Accordingly, there is no reason why natural variability should not be discarded for long-term climate projections. This can simplify considerably the calculation of forcing of climate agents and minimize potential noise and errors associated with the natural variability. The errors propagate with time thus limiting reliable calculations of distant past or future climates.

In this manuscript, calculation of climate forcing does not utilize computer simulations; it is based on the mathematical derivation of climate forcing function resulting from greenhouse gas emissions and accounting for emissions time. Long-term climate calculations are considered and the impact of natural variability is neglected. The long-term impact of forcing caused by short-lived climate agents on the other hand is not fully understood, and, therefore, they are not considered. Only the average forcing of greenhouse gases is calculated, which is approximately equal to that of carbon dioxide. The calculated forcing and other climate parameters using this approach agrees reasonably with observations presented in the IPCC Fifth Assessment Report (AR5) and the work of other climate researchers. Because the atmospheric air resembles an ideal gas, the calculated natural and anthropogenic forcings by the derived mathematical formulas are expected to be reasonably accurate.

Related Model

The solar energy exchanged with the earth is equal to the difference between incoming solar radiations and outgoing energy reflected and radiated by the earth’s surface and atmosphere. This energy exchanged is removed by evaporating surface water. At the conclusion of one revolution of the earth around the sun, or an average year, the change in the enthalpy and potential energy of the atmospheric air and that of surface is negligible; it is a repeatable cycle. The only change that occurs in the process is that water vapor phase changes from water at surface to water vapor in the air then back to surface water as precipitation. Therefore, the solar energy exchanged with the earth (surface plus atmospheric air) is equal to the latent heat of water evaporation, which is equal to that of condensation of precipitation.

The atmospheric air covers the earth as a blanket through which the latent heat of water vapor condensation must pass first before being radiated to outer space. Therefore atmospheric air chemistry does affect the amount of heat radiated. Based on [14], p. 255 and 259, and [15], Equation 12.68, the energy radiated from the atmospheric air can be represented by the following equality:

$$Qr = \varepsilon\ \sigma\ To^4 \tag{1}$$

Where

Qr =Energy radiated from the atmospheric air to outer space, W m^{-2}.
ε =Emissivity of the atmospheric air, dimensionless, approximately equal to unity.
σ =Stefan-Boltzmann Constant, 5.67 x 10^{-8}, W m_{-2} °K^{-4}.
To =Average mesopause temperature≈196 °K, Table 1 of [16].

The calculated energy radiated by the atmosphere using Equation (1) is 83.7 W m^{-2}, which does not include energy radiated by clouds or particulate matters entrained in the atmospheric air. This energy is equal to that of the latent heat of condensation of water vapor, or precipitation, which is also equal to the energy absorbed by the earth. The observed rainfall is approximately equal to one (1) meter annually, [17]. The equivalent latent heat of condensation of water vapor is nearly 78.1 W m^{-2}. For the latent heat, [18] suggests 80-88 W m^{-2}, and [19] proposes 80 W m^{-2}. The calculated radiated energy by Equation (1) is reasonably close to the observed energy absorbed by the earth and those calculated or proposed by others. Therefore, Equation (1) is acceptable and can be used to calculate the energy radiated from the atmospheric air to outer space. This radiated energy is variable, depending on the temperature profile of the atmosphere, which varies with air chemistry. A larger air molecular weight yields smaller lapse rate and smaller temperature of the upper atmosphere. Therefore, the atmospheric air radiates less energy and the surface of the earth accumulates heat. This is the effect of carbon dioxide emitted; its molecular weight is greater than that of air.

The lapse rate equation of the atmospheric air follows:

$$T-T^{*}=-(g/R)[(\gamma-1)/\gamma]\ M\ (Z-Z^{*})$$

Where

T =Temperature of the atmospheric air in ^{0}K at a given height, Z, in kilometers.
T* =Temperature of the atmospheric air in ^{0}K at a reference height, Z*, in kilometers.
g =Gravity acceleration, 9.8 m s^{-2}.
R =Gas constant, 8.315 kJ $kmol^{-1}$ °K^{-1}.
γ =Ratio of specific heats of air, 1.4, dimensionless.
M = Molecular weight of the air mixture, 28.8.

The ratio of specific heats does not vary greatly between surface and mesopause and its average value of 1.4 can be used throughout the atmospheric air. The molecular weight of the air mixture, on the other hand, is variable because greenhouse gases have greater molecular weight than that of air. The molecular weight of greenhouse gases is approximately equal to that of carbon dioxide, 44, whereas the molecular weight of air is 28.8. At surface, the reference height, Z*, is equal to zero, and at the mesopause, Z-Z* is equal to, hm, and T=To. Where, hm, is the average height of the mesopause in kilometers and, To, is the average temperature of the mesopause in °K. The height of mesopause, hm, varies as an effect of the climate change energy balance; it is time dependent and does not vary with carbon dioxide emissions.

The lapse rate equation is valid in the lower troposphere and does not apply for the entire atmosphere. However, because of the infinitesimal variations of the molecular weight of the atmospheric air mixture with carbon dioxide emissions, it can be demonstrated that the differential of air temperature of the lapse rate equation with respect to variations in the concentration of carbon dioxide emissions applies throughout the atmosphere. In the mesopause region, the atmospheric air continues to behave as an ideal gas surrounded by empty space. The differential of the temperature at the mesopause, To, with respect to a small increase in the concentration of carbon dioxide with time is equal to $dTo=-(g/R)\ hm\ [(\gamma-1)/\gamma]\ dM$. Where dM is the increase in the molecular weight of air following a small increase of carbon dioxide emissions. Based on [20], Equations 4-161 and 4-162, the following applies for the atmospheric air mixture:

$$M=M_{Co2} \times y_{Co2}+M_{air} \times y_{air}$$

$$y_{Co2}+y_{air}=1$$

$$dM=(M_{Co2}-M_{air}) \, dy_{Co2}$$

$$dy_{Co2}=d(ppmv) \times 10^{-6}$$

Where

M =Average molecular weight of the air mixture.
M_{Co2} =Molecular weight of carbon dioxide, 44.
M_{air} =Molecular weight of air, 28.8.
y_{Co2} =Molar or volumetric fraction of carbon dioxide in the air mixture.
y_{air} =Molar or volumetric fraction of air.
dy_{Co2} =Infinitesimal change of the molar fraction of carbon dioxide in the air mixture.
d(ppmv)=Infinitesimal variations of the concentration of carbon dioxide in the air mixture in parts per million by volume, ppmv.

The concentration of carbon dioxide in the air mixture prior to the onset of carbon dioxide emissions is equal to 280 parts per million by volume, ppmv. This is assumed to be the beginning of the Industrial Revolution, 1750, which is a reasonable assumption based on the IPCC findings. Thereafter, the concentration of carbon dioxide increased and the variations, d(ppmv), are positive. d(ppmv)=ppmv-280. The change, dTo, is thus equal to the decrease in mesopause temperature in ^{0}K following an increase in the concentration of carbon dioxide in the atmosphere by d(ppmv). The average height of the mesopause, hm, is approximately equal to 96 kilometers based on Table 1 of [16]. Substituting the values, $dM=(44-28.8) \times d(ppmv) \times 10^{-6}=1.52 \times 10^{-5}$ (ppmv-280). Therefore, the reduction in the temperature of the mesopause $dTo=-(g/R)\, hm\, [(\gamma-1)/\gamma]\, dM$ is equal to $-(9.8/8.315) \times 96 \times [(1.4-1)/1.4] \times 1.52\ 10^{-5} \times$ (ppmv-280), and

$$dTo=-4.91 \times 10^{-4} \times (ppmv-280) \tag{2}$$

A decrease in the temperature of the mesopause, To, resulting from greenhouse gas emissions must necessarily reduce the energy radiated by the atmospheric air to outer space based on equation (1). The reduction in the energy radiated must also be equal to that accumulated in the surface, which is typically expressed in forcing, W m^{-2}. The infinitesimal value of carbon dioxide forcing or anthropogenic forcing, N(i), for a small and discrete period of time (i) can be obtained by the following equation:

$$N(i)=dQr=4\sigma\, To^3\, dToi/2 \tag{3}$$

The division by 2 in Equation (3) is required in order to calculate an average forcing during the period of time (i) under consideration. In practice, the value of (i) is one year at the end of which the concentration of carbon dioxide is typically provided. The corresponding calculated N(i), if used, would be the maximum forcing value for the entire year, which is incorrect. The average value is the correct figure to be used in the calculations and the division by 2 in Equation (3) is warranted. Equation (3) yields to the following Equation (4):

$$N(i)=-4\sigma\, To^3 \times 4.91 \times 10^{-4}\, (ppmvi-280)/2=-4.19 \times 10^{-4} \times (ppmvi-280) \tag{4}$$

The forcing at surface has an opposing sign to that of N(i) calculated by Equation (4). The net forcing at surface resulting from carbon dioxide emissions at the conclusion of, n, years is equal to the summation of -N(i) between zero and n:

$$N=\sum_{i=0}^{n} -N(i)=\sum_{i=0}^{n} 4.19 \times 10^{-4}\, [ppmvi-280] \tag{5}$$

This Equation (5) is a mathematical expression of the anthropogenic forcing at surface, where, ppmvi, is the annual concentration of greenhouse gases in the atmospheric air for the year (i) in parts per million by volume, ppmv, and 280 is the concentration of greenhouse gases in ppmv of the base-line year, 1750.

For past natural climates, the concentration of carbon dioxide in the air varied virtually linearly with time. The summation of Equation (5) simplifies to an arithmetic series as follows:

$$N=\overset{n}{\underset{0}{\textstyle\sum_i}} 4.19 \times 10^{-4}\,[ppmvi-280]=\overset{n}{\underset{0}{\textstyle\sum_i}} 4.19 \times 10^{-4} \times d(ppmvi)=\overset{n}{\underset{0}{\textstyle\sum_i}} 4.19 \times 10^{-4}(i/n)\,\Delta(ppmv)$$

$$N=[4.19 \times 10^{-4}\,\Delta(ppmv)/n]\overset{n}{\underset{0}{\textstyle\sum_i}} i=[4.19 \times 10^{-4} \times \Delta(ppmv)/n] \times (0+n) \times n/2$$

$$N=2.10 \times 10^{-4} \times \Delta(ppmv) \times n \qquad (5.1)$$

Equation (5.1) is a mathematical expression of the natural forcing, where Δ(ppmv) is the total change of the concentration of carbon dioxide in parts per million by volume during climate transformation time, n, in years and d(ppmvi) is the annual change of the concentration of carbon dioxide in parts per million by volume. Because of the linearity of carbon dioxide emissions with time, d(ppmvi)=(i/n) x Δ(ppmv).

Data and Methods

There are sufficient data available to confirm the correctness of the Related Model and validate the derived equations (5) and (5.1). The data used for this work are published in peer reviewed journals. The Intergovernmental Panel on Climate Change (IPCC) website provides assessment reports relative to the current warming trend. They include valuable data and information to compare with the calculations. The most recent report is the Fifth Assessment Report (AR5) of 2013, [10]. These reports are based on peer reviewed publications by researchers from all over the world. They provide additional references that can be referred to for further reading on the subject of forcings of climate agents.

Equation (5) provides a simple mathematical formula for calculating forcing of carbon dioxide emissions, yet accurate as will be demonstrated. Carbon dioxide emission is the prevailing anthropogenic climate agent, which can be obtained from the Keeling Curve. The Curve plots carbon dioxide concentration in the atmosphere in parts per million by volume, ppmv, with time starting 1960. Between 1750, the base-line year, and 1960, a linear relationship is assumed. From this prepared data, the calculation of anthropogenic forcing is straight forward by employing Equation (5). Results are tabulated in Table 1.

The forcing of past climates can be calculated by knowing the change in the concentration of carbon dioxide emissions in the atmosphere and emission time, Equation (5.1). They are available in Antarctica Ice Core Data. It should be noted that the age of sample ice may differ from the age of sample air by several hundred years. For more accuracy, emission time is considered to be equal to the average of those for carbon dioxide and surface temperature. Between 18 000 BP and 13 750 BP, carbon dioxide concentration increased by approximately 55 ppmv. Therefore n=18 000-13 750=4 250 years, Δppmv= 55, and climate forcing N=2.10 x 10^{-4} x 55 x 4 250=49.10 W m^{-2}.Carbon dioxide forcing for the present and past climate trends can be approximately calculated by scaling up or down trend durations in years and variation of carbon dioxide emissions in parts per million by volume. For example, climate forcing through 1960 can be calculated from the warming campaign that occurred between 18 000 BP and 13 750 BP as follows: N(1960)=49.10 x (1960-1750) x (316-280)/(4 250 x 55)=1.59 W m^{-2}. Through the year 2011, climate forcing is equal to 49.10 x (2011-1750) x (390.5-280)/(4 250 x 55)=6.06 W m^{-2}. This calculated value of forcing is higher than observed, 2.29 [1.13 to 3.33] W m^{-2} based on [10], because the present warming trend is not similar to that which occurred between 18 000 BP and 13 750 BP. The present warming trend is not linear with time and the maximum concentration of carbon dioxide, 390.5, is used for the entire period of time. This is incorrect for it would yield higher forcing values than actual values. The actual climate forcing is approximately equal to the average 6.06/2=3.03 W m^{-2}. This scaled-down forcing agrees with the observed and calculated climate forcing in Table 1.

TABLE 1. CALCULATED ANTHROPOGENIC FORCING, N IN W M^{-2}, FOR THE PRESENT WARMING TREND. THE CONCENTRATION, PPMV, IS PARTS PER MILLION BY VOLUME OF CARBON DIOXIDE IN THE ATMOSPHERIC AIR. CARBON DIOXIDE (CO2) ANNUAL TREND IS ASSUMED TO INCREASE BY 0.63% BASED ON THE U. S. ENERGY INFORMATION ADMINISTRATION.

Year	CO2 ppmv	N W m^{-2}	Year	CO2 ppmv	N W m^{-2}
1750	280.0	0.00	2011	390.5	3.06
1901	305.9	0.82	2012	393.0	3.11
1950	314.3	1.44	2015	400.5	3.26
1960	316.0	1.59	2020	413.0	3.53
1961	316.9	1.61	2030	440.0	4.11
1970	325.0	1.76	2040	468.0	4.84
1971	326.3	1.78	2050	499.0	5.70
1980	338.0	1.98	2060	531.0	6.70
1981	339.5	2.01	2068	559.0	7.59
1990	353.0	2.26	2069	562.5	7.71
1991	354.5	2.29	2070	566.0	7.83
2000	368.0	2.60	2080	602.0	9.11
2001	370.0	2.64	2090	641.0	10.55
2010	388.0	3.02	2100	683.0	12.16

Aerosols forcing of volcanic events can be computed using the derived equations as well, depending on the observed climate parameter recorded or considered. The Pinatubo volcanic eruption is reasonably documented in the literature. Based on figure 1 of [12], the average response of mesopause temperature increased by 11 ^{0}K in approximately one year following aerosols development. This observation is sufficient to estimate aerosols forcing using the derived equations. Equation (3) is a suitable staring point. On a monthly basis, the variable, i, is equal to one month instead of one year and $N(i)=+2\sigma\ To^3\ dToi/12$. The sign is positive because mesopause temperature increased in the first year as figure 1 of [12] shows. Therefore, the forcing, N, at surface has an opposing sign and it is equal to the following summation:

$$N=-\sum_{i=0}^{n} N(i)=-\sum_{i=0}^{n} \sigma\ To^3\ dToi/6$$

From Fig. 1 of [12], it is reasonable to consider that the monthly change in the temperature of the mesopause, dToi, is linear with time, or $dToi=(i/n) \times \Delta To$. Where, i, is the month in consideration, and, n, is the total number of months, 12, during which the temperature of the mesopause assumed a maximum value of ΔTo. Consequently, the forcing can be calculated by knowing only the average response, $\Delta To=11\ ^{0}K$, at the conclusion of n=12 months as follows:

$$N=-\sum_{i=0}^{n} \sigma\ To^3\ dToi/6=-(1/6)\sum_{i=0}^{n} \sigma\ To^3\ (i/n)\ \Delta To=-(1/6) \times (\sigma\ To^3/n)\ \Delta To \sum_{i=0}^{n} i$$

$$N=-(1/6)\ (\sigma\ To^3/n) \times \Delta To \times (0+n) \times n/2=-(1/12)\ (\sigma\ To^3) \times \Delta To \times 12$$

$$N=-\sigma\ To^3\ \Delta To$$

$$N=-5.67 \times 10^{-8} \times 196^3 \times 11=-4.70\ W\ m^{-2}.$$

Discussion and Conclusions

Many climate related publications suggest that radiation occurs at all levels of the mass of atmospheric air. This is inconsistent with the basic physics of radiation and published literature. Engineering reference books are based on successful applications, and convection heat transfer only exists within the mass of air, [20]. The lapse rate equation is an equality between potential energy and enthalpy of an air parcel, no radiation terms exist in the equation. Radiation heat transfer is by definition a surface phenomenon based on which radiation equations were derived, [15]. Landmark climate related publications such as [14] consider radiation to occur mainly at the external layers of the atmosphere interfacing with the surrounding empty space. This landmark publication uses Equation (1) to calculate atmospheric air radiation. For these reasons and calculation agreement with observed energy radiated by the atmospheric air, Equation (1) is valid as used. In addition, typical textbooks of physics and chemistry, Dalton's law, engineering references such as [20], and practical applications reveal that carbon dioxide emitted into the atmospheric air does not behave as a separate entity. It is part of the atmospheric air mixture and behaves as such. If carbon dioxide emissions absorb more solar energy, it is the atmospheric air mixture as a whole that absorbs more energy.

Carbon dioxide emissions increase the molecular weight of the air mixture because its molecular weight is greater than that of air. This induces a thermodynamic transformation where the temperature profile of the atmospheric air decreases with the emissions. An increase in the concentration of carbon dioxide in the atmospheric air by 100 ppmv increases the molecular weight of the air mixture by 0.005%. Mesopause temperature decreases by 0.05 ^{0}K. The outcome of this transformation is radiated energy reduction to outer space and heat accumulation in the surface. Although carbon dioxide emissions are small in laboratory terms, their effect is amplified by the large surface area of the atmosphere and emission time.

The effect of carbon dioxide emissions is time dependent as Equation (5), Equation (5.1), their validation, and observations demonstrate. This holds true for anthropogenic and natural forcings. The derived equations in this manuscript account for this fundamental argument and the independent variable of the carbon dioxide forcing function. Furthermore, past and present carbon dioxide forcings can be scaled from one another by considering variations of the concentration of carbon dioxide and emission time. These indicate that emission time and carbon cycle, or life on earth, are the only forcing function independent variables. Weather and natural variability should therefore be discarded for long-term climate projection. This conclusion is confirmed by observations, [10, 11].

The IPCC reports are based on research works published in scientific journals and they include direct and remote measurements of climate parameters of the atmosphere and surface. These can be used as a basis for comparing the observed climate change parameters with those calculated in this manuscript. Reference [10] estimates anthropogenic forcing in Chapter 8, under Sect. 8.5.2, Time Evolution of Historical Forcing. The global mean forcing is 0.57 [0.29 to 0.85] for 1950, 1.25 [0.64 to 1.86] for 1980, and 2.29 [1.13 to 3.33] for 2011. The calculated average anthropogenic forcing in this work for the periods of time in consideration are 1.44 for 1950, 1.98 for 1980, and 3.06 for 2011 presented in Table 1. The calculated forcing for 1950 is greater than that used by the IPCC. This is expected because the annual carbon dioxide emissions prior to 1960 are unavailable and a linear relationship is assumed between 1750 and 1960 for the calculations. This assumption does not necessarily reflect the actual scenario of carbon dioxide emissions in this period of time. Extrapolation of the Keeling Curve suggests that carbon dioxide emissions were not linear with time; they had values below the assumed linear trend. Consequently, the forcing in 1950 must be less than the calculated 1.44 W m^{-2}. The calculated climate forcing for 1980 is close to the upper limit of the IPCC range and that for 2011 is reasonably in agreement. It is obvious that if the actual carbon dioxide trend prior to 1960 is used, the calculated forcing values will agree more with observations. Figure 8.20 of [10] shows that the total anthropogenic forcing between 1980 and 2011 is about 1.0 W m^{-2}. This is in agreement with the calculated forcing of 1.08 W m^{-2}, Table 1. The present annual energy accumulation is approximately equal to 0.05 W m^{-2}, Section 2.3.2, of Chapter 2 of [10], and the calculated present trend between 2011 and 2012 in Table 1 is 0.05 W m^{-2}, they are in good agreement.

A reduction in the energy radiated to outer space for the current warming trend is observed. Based on [3], the data for the top of the atmosphere irradiance are decreasing at the rate of 0.05 W m^{-2} annually. This is in very good agreement with the theoretically calculated decreasing trend of the energy radiated to outer space, 0.05 W m^{-2}. Also, it agrees with the observed annual energy increasing trend of 0.05 W m^{-2} as reported by the IPCC.

The Pinatubo volcanic eruption is an opportunity to validate climate calculations because of the separate nature of the event. The calculated aerosols forcing following Pinatubo volcanic eruption of -4.7 W m^{-2} agrees with observations and the work of others. Reference [21] calculated Pinatubo forcing to be on the order of -5.0 W m^{-2}; [22] obtained -3.0 W m^{-2} for Pinatubo; and [10] estimate of Pinatubo forcing is -3.0 W m^{-2}, Section TS.3.5, Radiative Forcing From Natural Drivers of Climate Change. Clearly, the observed and calculated figures are of the same order of magnitude, they differ by data sources.

Based on these agreements with present and past observations and the work of others, it is fair to conclude that the derived equations (5) and (5.1) are acceptable. The equations can therefore be used to project climate parameters.

Because the atmospheric air resembles an ideal gas, the source equations used to derive these climate equations apply and accuracy is anticipated. This has relevant merit at the societal level for the expected surface temperature and sea level rise with carbon dioxide emissions can be determined accurately.

The advantages of this mathematical approach of calculating natural and anthropogenic forcings are simplicity and accuracy. Long-term projection of anthropogenic forcing should not consider natural or internal variability such as volcanic eruptions or ENSO because their effects cancel out with time. These variabilities typically impart uncertainty to computer simulations, especially for long term projection where errors propagate with time thus limiting the length of climate projection. The uncertainty is evident in Section 9.2.3 of Chapter 9 of the IPCC Fifth Assessment Report (AR5).

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