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4 Manuscript title: Deforestation and land farming as regulators of population size and climate  
5 Short Title: Deforestation

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46 **Abstract**

47

48 This work reveals that deforestation and farming of the cleared land may be considered as  
49 regulators of the size of world population and climate. When the green matter is cleared, it  
50 decomposes, oxidizes, and releases to the climate the heat of carbon conversion to carbon  
51 dioxide. The farmed land provides carrying capacity that sustains population growth. As world  
52 population grows, energy consumption increases. Most of this energy has been fossil fuel based,  
53 and the heat released is gained by the climate. Warming of the surface and population growth are  
54 thus processes that depend on land clearing and conversion into farmland. If deforestation  
55 increases, the size of the farmland increases, population size increases, and the surface  
56 accumulates heat and vice versa. Therefore, analyses of the correlation between deforestation,  
57 population growth, and climate change may have merit for research and society. Through 1990,  
58 deforestation has contributed at least 36% of the total heat added to the surface. Therefore,  
59 deforestation contribution to climate change is substantial. At the current rate of deforestation,  
60 the world population may approach 14.5 billion by the year 2100. Should deforestation decrease  
61 and approach zero by then, the world population may steady at nearly 10.3 billion instead.

62

63 Keywords: Deforestation; Photosynthesis; World Population Growth; Energy Production;  
64 Climate Change

65

66 **1. Introduction**

67 Food and Agriculture Organization of the United Nations (FAO 2016a) reveals that from  
68 early human history the area of land covered by forests has decreased by nearly 50%. “There has  
69 been a relationship between population growth, increased demand for agricultural land, and  
70 forest loss dates back thousands of years. Forests have sometimes re-established naturally as  
71 deforestation pressures have eased. Archeological and historical records reveal that the decrease  
72 in forest size is associated to population growth and demand for crops and grazing land. In  
73 locations of the world that had experienced severe disease and population decline, forests re-  
74 grew again.” The historical and archeological records potentially correlate deforestation with  
75 population growth. Deforestation and farming of the cleared land provide resources that sustain  
76 world population. When the world population increases in size, demand for energy increases as  
77 well (EIA 2016). Therefore, investigating drivers and regulators of population growth at the  
78 global level are important for the objectives of this manuscript.

79 Statistical models to calculate and project populations have been successfully used. Jones and  
80 Tertilt (2006) summarize the history of fertility in the U.S.A. since 1826, its relationship with  
81 economy and social factors, as well as statistical models used to calculate population size.  
82 Similar statistical methodologies have been adopted by United Nations UN (2019) for world  
83 populations. These methodologies appear to assume human factors as regulators of human  
84 population, and human population is thus an independent variable. Conversely, this work reveals  
85 that world population may be calculated and projected reasonably by considering only  
86 deforestation and farming of the cleared land as a variable. The applicable laws of population  
87 growth to plants and organisms appear to be applicable to humans as well. They all need  
88 nutrients to multiply and human population is therefore a dependent variable instead.

89 Deforestation is the opposite process of photosynthesis, in that the chemical energy stored in  
90 plants' tissues is converted into climate heat. It raises surface temperature. Farming of the  
91 cleared land increases the carrying capacity of the farmland available and world population  
92 grows. This growth requires energy to be maintained, and most of this energy has been fossil fuel  
93 based. Deforestation and fossil fuel burning produce heat and carbon dioxide that interact with  
94 the climate in accordance with the laws of thermodynamics. The outcome of this interaction is  
95 that all of the heat of deforestation and fossil fuel burning accumulates in the surface (Swedan  
96 2019). Also, the methodology presented in (IPCC 2013) uses the total variation in the  
97 concentration of carbon dioxide in the atmosphere as the basis to calculate changes in the  
98 climate. The methodology utilizes the Radiative Forcing concept, discussed in Chapter 8 of this  
99 reference, and the resulting radiative forcings are reported in  $W m^{-2}$ . There appears to be no  
100 direct mathematical relationship that calculates the heat of deforestation at this time. It is thus  
101 derived in this manuscript. In addition, the radiative methodology used in the Intergovernmental  
102 Panel on Climate Change reports assumes that climate agents impose radiative imbalance at the  
103 top of the atmosphere. Such a methodology may only provide an overall anthropogenic forcing,  
104 and biosphere thermodynamic interactions may not be captured by this methodology. Limited  
105 information may thus be obtained by the radiative model. The manuscript on the other hand  
106 discusses biosphere thermodynamic and heat exchange interactions as a result of human  
107 development and civilized structure. The information that may be obtained by the proposed  
108 methodology are many including but not limited to world population growth, deforestation,  
109 energy production, efficiency of photosynthesis, and surface greening.

110 This work reveals that there exist relationships between deforestation and farming of the  
111 cleared land, heat of deforestation, population size, energy production, and surface warming. The  
112 independent variable is deforestation and farming of the cleared land. Dedicated mathematical  
113 relationships to calculate the heat of deforestation and size of world population, are, therefore,  
114 derived and validated in this manuscript. They are found to be in agreement with observations.  
115 Because of the breadth and multidisciplinary nature of this work, a section "Symbols and  
116 Abbreviations" is appended to the end of this paper. The intended meanings of parameters and  
117 symbols are explained in this section.

118

## 119 **2. Background Information**

120

121 Long before the Industrial Revolution, deforestation was minimal and population size was  
122 relatively small based on information provided by (FAO 2016a; FAO 2016b). Deforestation then  
123 increased by nearly 0.2% annually through the first half of the twentieth century to nearly 50% at  
124 the present time. The world population increased as well. In the nineties, deforestation began to  
125 decrease, and population growth rate has decreased. By 2010, deforestation was estimated at  
126 0.08% annually and it is expected to decrease with time. So is population growth projection by  
127 (UN 2019).

128 The United Nations (UN 2019) analysis attributes the observed and projected decrease in  
129 population growth rate to reduced levels of fertility. Population projection utilizes demographic  
130 and statistical methodologies. The decrease in fertility is attributed to "human development  
131 factors such as reduction in child mortality, increased level of education particularly for women  
132 and girls, increased urbanization, expanded access to reproductive health care, women's  
133 empowerment and growing labor force participation," and other similar human and social

134 factors. The observed decrease in deforestation and size of farmland trends are not mentioned as  
135 causes of the observed reduction in world population growth rate.

136 Apparently, the methodologies adopted assume growth of human population to be an  
137 independent variable, strictly driven by human related factors. This is likely to be true and may  
138 continue to be applicable for specific geographic locations or regions of the world. However, at  
139 the global level population growth resembles growth of other organisms and plants as this work  
140 reveals. The basic natural laws applicable to organisms and plants appear to apply to humans as  
141 well. Textbooks of ecology, for example (Cain et al. 2014), reveal that populations can grow  
142 when conditions are favorable. When environmental conditions are ideal, populations grow  
143 exponentially; however, the growth cannot continue indefinitely. There is a limited carrying  
144 capacity for each ecosystem that population size may not exceed. Population growth increases  
145 population density and species have to compete for space, nutrients, and other ecological and  
146 environmental parameters. The environmental conditions become less favorable with time and  
147 population growth deviates from that of exponential growth. It tends asymptotically to the  
148 carrying capacity, defined as the maximum population size an ecosystem can sustain. This  
149 growth pattern is typically referred to as logistic growth. For this growth, population density may  
150 be considered as a population regulator. Logistic growth curve fits have been used to calculate  
151 specie populations and human populations of geographic locations (Pearl and Reed 1920).

152 Until the twentieth century, most humans lived in rural areas (UNESCO 2019) where  
153 population density was not a major factor. Presently, the majority of the world lives in cities  
154 based on this reference, and the urbanization trend is projected to continue. Population density  
155 may thus be a factor in the observed decrease in population growth rate. However, humans have  
156 the ability to relocate and emigrate for better environmental conditions. While urban population  
157 density may have a role in the observed decrease in population growth rate, it may not be as  
158 definitive as for plants or organisms. For these, the carrying capacity of their immediate  
159 ecosystem is a limiting factor. For humans, the entire biosphere is their ecosystem. Manufactured  
160 and food products may be shipped around the world, and humans can leave a poor immediate  
161 ecosystem for a better one. Human population is ultimately dictated by how much farmland is  
162 available in the biosphere. The farmland supplies nutrients and provides carrying capacity.  
163 Consequently, population size is assumed to be a dependent parameter, mainly on the available  
164 farmland and population function is derived. The calculated populations are compared with  
165 observations and (UN 2019) projections and found to be in agreement.

166 Energy consumption is associated with population size, and this energy has been provided  
167 mostly by burning fossil fuels. Therefore, it is reasonable to associate an increase in energy  
168 consumption to deforestation and farming of the cleared land. Deforestation on the other hand is  
169 the opposite process of photosynthesis; the green matter oxidizes, decomposes, and combusts,  
170 and the heat of carbon conversion to carbon dioxide is released to the surroundings. Based on  
171 this understanding, the total heat of deforestation and energy consumption is dependent on  
172 deforestation. Deforestation is thus the independent parameter and regulator of world population  
173 and climate. Accordingly, mathematical relationships between deforestation, world population,  
174 and climate exist; they are derived in this manuscript. The significance of the derived equations  
175 is discussed and their applications illustrated. The calculated heat added to the surface is  
176 compared with observations and found to be in agreement as well.

177  
178

### 179 **3. Data**

180  
181 The data used for this work are available in the public domain, accessible by the links  
182 provided under references. Required for this work are green matter growth relationships and  
183 efficiency of seasonal photosynthesis. They are calculated from available seasonal variations of  
184 carbon dioxide in a report prepared by U.S. Environmental Protection Agency (EPA 2012). In  
185 order to produce Table 1 and figures 1 through 4, past and present history of deforestation is  
186 summarized in reports prepared by Food and Agriculture Organization of the United Nations  
187 (FAO 2016a; FAO 2016b). For comparison with the calculations, observed world populations  
188 are obtained from United Nations report (UN 2019). The Intergovernmental Panel on Climate  
189 Change (IPCC) summarizes changes in the climate and provides a historical record of the  
190 changes as well as forcings for comparison with the calculated heat added to the surface. Their  
191 most recent report (IPCC 2013) is used as a source of data. Energy consumption of the world  
192 population is an integral part of this work; it is compiled since 1990 by the Energy Information  
193 Administration (EIA 2016). Reference and textbooks such as Perry and Green (1984) and  
194 Jørgensen and Svirezhev (2004) are used to obtain heat of decay, combustion, and oxidization of  
195 the green matter and compare calculated photosynthesis efficiency with those calculated by  
196 others.

### 197 **4. Method**

198  
199  
200 The geological record reveals that changes in climate occurred long before humans became  
201 influential in the biosphere. Warming and cooling cycles existed several hundred thousand years  
202 ago (Petite et al. 1999). Causes of these changes are attributed to the inherent tendency of green  
203 matter to multiply and the resulting variations in its size (Swedan 2019). The size of plants and  
204 vegetation has been variable with human development in the forms of deforestation, and  
205 apparently, surface greening (FAO 2016b; Zaichun et al. 2016). Variation in the size of green  
206 matter induces thermodynamic transformations in the biosphere and human activities are thus an  
207 integral part of biosphere thermodynamics. Therefore, the model used in this work assumes that  
208 humans and their surrounding biosphere are an ecosystem. Unlike primitive forms of life,  
209 humans can travel and ship manufactured and food products around the ecosystem. They are not  
210 limited by space or time. For example, farm products in developing countries find their way to  
211 consumers in industrialized countries. Similarly, manufactured products in industrialized  
212 countries are sold in developing countries. Shipping time of goods has so much decreased to the  
213 point that it is reasonable to assume that changes in the biosphere due to humans are global in  
214 nature and practically immediate. At the center of these goods are basic forest and agricultural  
215 products. Kissinger et al. (2012) reveal that most of these goods produced in the cleared lands are  
216 for export. Farmland may thus be considered to belong to the world population, and deforestation  
217 and its conversion into farmland increase biosphere carrying capacity.

218 Therefore, population size is assumed to be proportional to the total size of farmland available  
219 in the biosphere. The forests converted into farmland support additional world population. In  
220 turn, the population requires more energy for heating and air conditioning, lighting, appliances,  
221 infrastructure, transportation, recreation, and others. Most of this energy has been provided by  
222 burning fossil fuels since the Industrial Revolution, assumed 1750 by (IPCC 2013). This energy  
223 produces heat and carbon dioxide. Deforestation produces heat and carbon dioxide as well.

224 Together, deforestation and energy consumption add heat and carbon dioxide to the climate. The  
225 surface gains this heat as a result.

226 Based on this understanding, equations of heat of deforestation and world population size are  
227 derived and reduced to simple formulas so that the entire work may be reproduced by others.  
228 Basic calculation tools thus suffice. In this presentation a calculation spreadsheet, Microsoft  
229 Excel, is used to prepare Table 1 and figures 1 through 4. Given that the independent variables of  
230 the formulas may change with time as explained in the conclusion section, programming using a  
231 higher level language may be recommended for more accuracy.

232

## 233 **5. Theory and Model**

234

### 235 **5.1 Heat of Deforestation**

236

237 Equation (20) of Swedan (2019) will be used as the basis to derive equations of seasonal  
238 efficiency of photosynthesis, plant growth, and heat of deforestation. The equation follows:

239

$$240 \quad dn_G/n_G = (dppmv_{CO_2}/2)/ppmv_{CO_2} \quad (1)$$

241

242 Where

243

244  $dn_G$  = Average variation in the number of moles of the green matter, mol.

245  $n_G$  = Number of moles of the green matter in the biosphere, mol.

246  $dppmv_{CO_2}$  = Variation in the concentration of carbon dioxide in the atmosphere in parts per  
247 million by volume, ppmv.

248  $ppmv_{CO_2}$  = Concentration of carbon dioxide in the atmosphere in parts per million by volume,  
249 ppmv.

250

251 Plants grow annually and during the warming season. Their growth reaches maximum value  
252 toward the end of this season. Therefore, the maximum variation in the number of moles of the  
253 green matter  $dn_G$ , which is equal to  $2 \times dn_G$  is required for this analysis. The term  $dn_G/n_G$  may be  
254 thought of as the maximum value of seasonal efficiency of photosynthesis, which is equal to  
255  $dppmv_{CO_2}/ppmv_{CO_2}$ . Consequently

256

$$257 \quad dn_G/n_G = \eta_{max} \quad (2)$$

$$258 \quad \eta_{max} = (dppmv_{CO_2})/ppmv_{CO_2} \quad (3)$$

259

260 Where

261

262  $dn_G$  = Maximum annual increase in the number of moles of the green matter, mol.

263  $\eta_{max}$  = Maximum value of seasonal or annual efficiency of photosynthesis, dimensionless.

264

265 Every year, a new green matter,  $dn_G$ , is added. Therefore, the total size of the green matter  
266 may be obtained by integration of Eq. (2) with time

267

268

269  $n_G$   
 270  $\int_{n_{G0}}^{n_G} \frac{dn_G}{n_G} = \int_{t_0}^t \eta_{\max} dt$  (4)  
 271  $n_{G0}$   $t_0$

272  
 273  $n_G = n_{G0} e^{[\eta_{\max} (t-t_0)]}$  (5)  
 274

275 Where

- 276  
 277  $n_G$  = Number of moles of the green matter at age, t, mol.  
 278  $n_{G0}$  = Number of moles of the green matter at a reference age,  $t_0$ , mol.  
 279 t = Age of the green matter, years.  
 280  $t_0$  = Reference age of the green matter at which its size is equal to  $n_{G0}$  moles, years.  
 281

282 Equation (5) gives the size of the green matter and vegetation as they age. However, plants do  
 283 not grow in size indefinitely; they die when they reach the end of their life. The dead wood  
 284 decays and decomposes, and this represents a reduction in carbon stock of forests. The report  
 285 (EPA 2012) reveals that if not for anthropogenic emission of carbon dioxide, variation in the  
 286 content of carbon dioxide in the atmosphere in a full year may be negligible. Or, natural decay of  
 287 biomasses is nearly equal to natural seasonal photosynthesis for small periods of time. This,  
 288 however, may not be true for extended periods of time, because green matter inherently tends to  
 289 increase in size when environmental conditions are favorable as discussed in the background  
 290 section. Therefore, if average efficiency of seasonal photosynthesis is equal to  $\eta$ , the annual  
 291 natural decay of the green matter is nearly equal to  $\eta \times n_{G0}$ . Consequently, the net size of the  
 292 green matter after subtracting natural wood decay follows

293  
 294  $n_{G_{\text{net}}} = n_{G0} e^{[\eta_{\max} (t-t_0)] - n_{G0} \eta (t-t_0)}$  (6)  
 295

296 Where  $n_{G_{\text{net}}}$  is the net size of the green matter, measured in moles, after the time  $t-t_0$  has  
 297 elapsed. The parameter,  $\eta$ , is average efficiency of photosynthesis. Deforestation is assumed to  
 298 decrease this net size of the green matter with time. The ratio between the annual size of the  
 299 cleared green matter and the total original size is equal to the annual deforestation fraction, d.  
 300 Therefore  $d = -dn_D/n_{G_{\text{net}}}$ . Where  $dn_D$  is equal to the number of moles of the cleared green matter  
 301 annually. The minus sign is a convention to denote deforestation. Multiplying both sides of Eq.  
 302 (6) by d

303  
 304  $dn_D = -d [n_{G0} e^{\{\eta_{\max} (t-t_0)\}} - n_{G0} \eta (t-t_0)]$  (7)  
 305

306 Where d is average annual deforestation fraction and,  $dn_D$ , is equal to the number of moles of  
 307 the green matter removed annually by deforestation. The removal decreases the chemical energy  
 308 of the green matter in the biosphere, and this decrease is equal to the heat of deforestation. The  
 309 heat of deforestation may be obtained by multiplying the number of moles of the green matter  
 310 removed by the heat of combustion of the green matter. For simplicity, this heat may be assumed

311 to be equal to that of carbon combustion. Multiplying both sides of Eq. (7) by the heat of carbon  
 312 combustion  $\Delta H_c$

$$313 \quad dQ_G = -\Delta H_c d [n_{G0} e^{\{\eta_{\max} (t-t_0)\}} - n_{G0} \eta (t-t_0)] \quad (8)$$

315  
 316 The term  $dQ_G$  represents the annual heat of deforestation. Integration both sides of Eq. (8)  
 317 between an initial year  $n_0$  and an arbitrary year  $n$ .

$$318 \quad Q_G - Q_{G0} = -\int_{n_0}^n \Delta H_c d [n_{G0} e^{\{\eta_{\max} (t-t_0)\}} - n_{G0} \eta (t-t_0)] dt \quad (9)$$

319  
 320 Where the quantity  $Q_{G0}$  represents the amount of chemical energy available in the green  
 321 matter at the initial year  $n_0$ .  $Q_G$  is the available chemical energy in the green matter after,  $n$ , years  
 322 of deforestation.  $Q_{G0}$  is greater than  $Q_G$ . Assuming that the reference age of the green matter,  $t_0$ ,  
 323 and the initial year,  $n_0$ , to be the same and equal to zero, the heat of deforestation follows

$$324 \quad Q_D = Q_G - Q_{G0} = -\Delta H_c d n_{G0} [(1/(\eta_{\max})) e^{\{\eta_{\max} (n)\}} - \eta n^2/2 - 1/(\eta_{\max})] \quad (10)$$

325  
 326 Where  $Q_D$  is equal to the total heat of deforestation after,  $n$ , years of deforestation. The value  
 327 of the heat of carbon combustion,  $\Delta H_c$ , is  $3.93 \times 10^5 \text{ J mol}^{-1}$  (Perry and Green 1984). The original  
 328 number of moles of the green matter,  $n_{G0}$ , before any deforestation, before 1750, may be  
 329 approximately estimated: Major deforestation may be assumed to have been initiated at about the  
 330 beginning of the Industrial Revolution. FAO (2016b) indicates that primary forests presently  
 331 cover approximately 30% of the land and average carbon content is nearly 74 tons per hectare.  
 332 Therefore, present inventory of green matter is  $1.11 \times 10^{22} \text{ J}$ . At 50% total deforestation and heat  
 333 of carbon combustion of  $3.93 \times 10^5 \text{ J mol}^{-1}$ ,  $n_{G0}$  is nearly equal to  $5.64 \times 10^{16}$  moles. Equation  
 334 (10) simplifies

$$335 \quad Q_D = -2.22 \times 10^{22} d [(1/(\eta_{\max})) e^{\{\eta_{\max} (n)\}} - \eta n^2/2 - 1/(\eta_{\max})] \quad (11)$$

336  
 337 Where  $Q_D$  is the total heat of deforestation in Joules after,  $n$ , years of deforestation at average  
 338 annual deforestation fraction  $d$ . The minus sign is a convention to indicate a decrease in the  
 339 chemical energy of the green matter. The surface gains the absolute value of  $Q_D$ .

## 340 5.2 Population Size

341  
 342 As discussed in the method section, the size of the world population is assumed to be  
 343 proportional to the total size of farmland available in the biosphere. The farmland provides the  
 344 carrying capacity. When new farmland is added, population grows exponentially until the  
 345 carrying capacity is satisfied. If the available size of farmland is equal to  $F$ , then

$$346 \quad dN/N = dF/F \quad (12)$$



354  $r_F = dF/dt$  (13)

355  $r = (1/N)dN/dt$  (14)

356

357 Where

358

359  $N$  =Population size, number of people.

360  $F$  =Size of the farmland,  $m^2$ .

361  $r_F$  =Rate of increase in the size of the farmland,  $m^2 t^{-1}$ .

362  $r$  =Population exponential growth rate,  $t^{-1}$ .

363

364 Equation (14) is available in typical textbooks of ecology, for example (Cain et al. 2014).

365 Equations (12) and (13) give

366

367  $dN/N = (r_F/F) dt$  (15)

368

369 Referring to Eq. (13), for  $dt$  equal to one, or one year,  $r_F$  is equal to the annual increase in the  
 370 size of farm land. From Eq. (13), the ratio  $(r_F/F)$  is thus equal to the fraction of the farmland  
 371 added annually,  $dF/F$ , which has been increasing. This fraction is not necessarily equal to the  
 372 annual deforestation fraction. However, because the current deforestation is at 50% of the  
 373 original forest size and deforestation data are reported with respect to this current deforestation,  
 374 then they are equal. The increase in the annual farmland fraction is equal to the increase in  
 375 annual deforestation fraction. The annual deforestation fraction is nearly equal to two times the  
 376 average annual deforestation fraction. Therefore,  $(r_F/F) = 2d$ , where  $d$  is average annual  
 377 deforestation fraction. Equation (15) and its integration yield

378

379  $dN/N = 2d dt$  (16)

380  $N = N_0 e^{[2d(t-t_0)]}$  (17)

381

382 Where,  $t$ , is an arbitrary time and,  $t_0$ , is an initial reference time.  $N$  is population size at the  
 383 time  $t$  and  $N_0$  is population size at the initial time  $t_0$ . The values of  $N$  may be thought of as  
 384 theoretical population that can be supported by the total farmland available at the time  $t$ . Also, it  
 385 may be thought of as the carrying capacity of the total land available at the time  $t$ . Within the  
 386 time,  $t-t_0$ , the total population available  $P(t)$  may be assumed to increase exponentially, and the  
 387 exponential rate of increase is equal to  $r$ . Referring to equations (14) and (15), for  $dt=1$ , or one  
 388 year,  $r = dN/N$  and Eq. (16) gives  $r = 2d$ . Therefore, the actual annual increase in population  
 389 follows

390

391  $dP(t) = rN = 2d N_0 e^{[2d(t-t_0)]}$  (18)

392

393 Where  $P(t)$  is the population function. The world population in any given year is the  
 394 cumulative sum of annual population increases, coordinated with farmland increases and  
 395 carrying capacities. For the first year,  $n=1$ , the change in population is  $dP(t)$ . For the second year,  
 396  $n=2$ , the total change is  $2dP(t)$ , and so forth because the carrying capacities of new farmlands add  
 397 cumulatively. Therefore

398  $P(n) = \int_{n_0}^n dP(t) = \int_{n_0}^n 2d N_0 e^{[2d(t-t_0)]} dt$  (19)  
 399  
 400  $P_0 = N_0$   
 401

402 Where  $P(n)$  is population size at an arbitrary year  $n$  and  $P_0$  is population size at a reference  
 403 initial year  $n_0$ . Assuming  $n_0=t_0=0$ , then  $P_0=N_0$  and population function  $P(n)$  follows  
 404

405  $P(n)=P_0[1+n e^{-(1/2 d n)} - (1/2 d) e^{-(1/2 d n)} + (1/2 d)]$  (20)  
 406

407 Where  $P(n)$  is population size calculated after  $n$  years have elapsed at average annual  
 408 deforestation fraction of  $d$ .  
 409

## 410 6. Significance and Application of the Derived Equations

411 Referring to Eq. (11), the average and maximum efficiencies of seasonal photosynthesis may  
 412 be assumed to be the same every year. Therefore, average annual deforestation,  $d$ , is the only  
 413 variable required to calculate the heat of deforestation. The equation reveals that this heat is time  
 414 dependent. Plants grow with time, equations (5) and (6). Therefore, for equal areas of forest  
 415 lands cleared in two different years  $n_1$  and  $n_2$ , where  $n_2$  is greater than  $n_1$ , the heat of  
 416 deforestation produced in  $n_2$  is greater than the heat of deforestation produced in  $n_1$ . The reason  
 417 is that plants grow in size in the time between  $n_1$  and  $n_2$ . Also, population size, Eq. (20), has only  
 418 deforestation as an independent parameter. Because energy consumption is a function of  
 419 population size, the total heat produced by deforestation and energy consumption is a function of  
 420 deforestation and farming of the cleared land.  
 421

422 Caution must be exercised when using equations (17) and (20) for world population. Unlike  
 423 organism or plant ecosystems, deforestation does not provide readily digestible nutrients to  
 424 people. As a result, the equations may not be used as traditional continuous functions to calculate  
 425 world population. Population size, Eq. (20), has to be applied for discrete and equal periods of  
 426 time having constant duration of years,  $n$ . The period of time cannot be selected arbitrarily, it  
 427 must be equal to the period of time of the life cycle of the processes comprising deforestation  
 428 and land clearing, land preparation for farming, farming of the prepared land; harvesting, storing,  
 429 and shipping of agricultural products, incorporation of the products in the food chain, family  
 430 planning and fertility decisions, and newborn registrations. The life cycle varies with agricultural  
 431 products such as grains, nuts, produce, or fruits, and may be estimated as the sum of the periods  
 432 of time required for each process of the life cycle. Alternatively, the duration of the life cycle,  $n$ ,  
 433 may be determined mathematically by using Eq. (20) and simple trial and error iterations to  
 434 match a narrow sample of observed world populations.

435 The life cycle and its duration do not appear to be addressed in the literature. It is thus  
 436 estimated to be between eight and 13 years, and ten years may be used as an average. Therefore,  
 437  $n=10=\text{constant}$  should be used for all of the discrete periods of time. Selecting discrete periods of  
 438 time smaller or greater than ten for the same initial reference population will be incorrect. For  
 439 illustration, assuming the world population of 1960 is used as a reference,  $P_0$ , at which  $t_0=n_0=0$ .  
 440 Using  $n=10$ , Eq. (20) correctly calculates world population every ten years in either direction.  
 441 For population growth between 1960 and 1970, deforestation of the previous decade, 1950-1960,

442 provides the carrying capacity, because deforestation between 1960 and 1970 has not yet  
443 provided carrying capacity. It takes nearly ten years for deforestation in 1960 to contribute to  
444 population growth. Using the reference year, 1960, to calculate population in 1969, will be  
445 incorrect, in that it does not account for the contribution of the resources of 1959. For 1969, the  
446 correct reference population has to be that of 1959. The same is true for longer periods than that  
447 of the life cycle: using population of 1960 as a reference to calculate population of 1971 will be  
448 incorrect as well, in that it assumes deforestation of 1961 has reached carrying capacity when in  
449 fact it has not. Therefore, by using known world populations in the record for any decade as  
450 reference years, past, present, and future populations may be calculated by knowing only average  
451 annual deforestation fractions. Accordingly, Table 1 and figures 3 and 4 are produced for  
452 evaluation and discussion.

453 To calculate the heat of deforestation using Eq. (11), the maximum value of annual seasonal  
454 efficiency of photosynthesis,  $\eta_{\max}$ , is required. EPA (2012) provides for 2011 average carbon  
455 dioxide content in the atmosphere of 391.67 ppmv and five (5) ppmv for seasonal carbon dioxide  
456 variation. Using Eq. (3), where  $\text{ppmv}_{\text{CO}_2}=391.67$  and  $\text{dppmv}_{\text{CO}_2}=5$ , the value of  $\eta_{\max}=0.013$ .  
457 The average value of seasonal efficiency of photosynthesis  $\eta=\eta_{\max}/2=0.007$ .

458 The observed anthropogenic forcings of the current warming trend are available in (IPCC  
459 2013). Figure 8.18 of Chapter 8 of this reference presents time evolution of anthropogenic  
460 forcing between 1750 and present time. Between 1750 and 1850, the forcing was small and  
461 negligible. Therefore, 1850 may be used as a representative reference year for comparison with  
462 the calculations. Because world energy consumption is unavailable before 1990, it is assumed to  
463 be linear through that year for simplicity. For 1990, Fig. 1-2 of EIA (2016) reveals that the world  
464 consumed nearly 350 quadrillion British Thermal Units (BTU), which is equal to the sum of  
465 energy consumed by OECD and Non-OECD countries. This is equivalent to  $3.70 \times 10^{20}$  J.  
466 Assuming world energy consumption in 1850 to be negligible, the total energy consumed  
467 between 1850 and 1990 is about  $2.59 \times 10^{22}$  J. Also, present total deforestation is nearly 50% of  
468 the original covered land, and annual average deforestation was approximately equal to 0.2%  
469 through 1990. For the period of time 1850-1990 and average annual deforestation of 0.2%, Eq.  
470 (11) yields heat of deforestation,  $Q_D$ , of nearly  $1.45 \times 10^{22}$  J. The total heat added to the surface  
471 by deforestation and energy consumption between 1850 and 1990 is thus equal to  $4.04 \times 10^{22}$  J.  
472 In Radiative Forcing terms, it is equal to  $2.52 \text{ Wm}^{-2}$ . The observed anthropogenic forcing for  
473 1990 is unavailable; however, it is 1.25[0.64 to 1.86] for 1980 and 2.29[1.13 to 3.33] for 2011  
474 based on Fig. 8.18 of (IPCC 2013). As a result, the observed forcing for 1990 must be between  
475  $1.86 \text{ Wm}^{-2}$  and  $3.33 \text{ Wm}^{-2}$ .

## 476 477 **7. Discussion**

478  
479 Kant and Berry (2005) suggest that deforestation and farming of the cleared land has  
480 increased the size of world population since farming was invented some 12 000 years ago.  
481 Generally, land farming followed small-scale and large-scale deforestations. Traditional small-  
482 scale farm products may be considered for local consumption that have increased and maintained  
483 local population size. However, this type of farming presently feeds a steadily decreasing rural  
484 population size as discussed in the background section. Therefore, it is reasonable to assume that  
485 at least some of the agricultural products produced by traditional farming compete with  
486 commercial agricultural products intended for global market. In addition, most of the

487 deforestation for this type of farming has already contributed to population growth.  
488 Consequently, any present type of deforestation and farming of the cleared land in the biosphere  
489 should be factored in the calculations.

490 Deforestation and life cycle duration,  $n$ , are required for Eq. (20). The values of average  
491 annual deforestation fractions are available in (FAO 2016b). However, the values used in the  
492 calculations must lag the observed values by the duration of the life cycle of agricultural  
493 products produced as discussed in Section 6. The cycle begins with deforestation and ends with  
494 the incorporation of agricultural products in the food chain and birth of newborns. Accordingly,  
495 the values of average annual deforestation fractions used in the calculations are presented  
496 graphically in Fig. 1.

497 As mentioned in Section 6, field data to estimate the life cycle do not appear to be available in  
498 the literature at this time. The life cycle may be between eight and 13 years. Also, the value of  $n$   
499 may be estimated from observed population growth, because this growth must reflect logistics of  
500 the life cycle. Therefore, Fig. 2 is prepared to summarize results of trial and error iterations to  
501 calculate life cycle duration  $n$ . The figure is related to observed populations sample between  
502 1970, reference year number zero, and 1980, year number ten. For 1971, year number one, the  
503 value of the iterated,  $n$ , so that the calculated population by equation (20) converges to the  
504 observed population is 3.1. For 1975, year number five,  $n$  is equal to 7.14. For 1980, year  
505 number ten,  $n=10$ . This pattern shown in Fig. 2 is repeatable every ten years for any human  
506 populations sample; apparently, it is characteristic of human population growth. Therefore, the  
507 value of  $n$  is nearly equal to ten and it is a constant. The world population may be calculated  
508 reasonably accurately every ten years measured from any given reference year using Eq. (20).

509 On these bases, Table 1 is prepared. The population of the year 1950 is used as a reference  
510 year for Eq. (20) and ten-year periods are used for  $n$ . The observed populations are obtained from  
511 (UN 2019). Populations are assumed to lag deforestations by ten years as explained in Section  
512 (6). For example, present average annual deforestation is nearly 0.08%. However, it is not shown  
513 or used in the calculations because its carrying capacity accrues in the future. The tabulated  
514 values of this Table 1 are graphically illustrated in Fig. 3. The figure reveals that the observed  
515 and calculated populations are practically in agreement. The maximum difference between  
516 calculated and observed populations is -1.27% or less as Table 1 indicates. Equation (20) is  
517 therefore assumed to represent population growth reasonably well. Consequently, Fig. 4 is  
518 produced to project population growth for two scenarios. In the first scenario, average annual  
519 deforestation is assumed to remain steady at its present rate of 0.08% annually. In the second  
520 scenario, deforestation is assumed to continue its decreasing trend linearly to negligible in 2100.

521

## 522 **8. Summary of Results and Conclusions**

523

524 The application part of Section 6 reveals that for climate calculations, the calculated forcing  
525 through 1990 is nearly  $2.52 \text{ Wm}^{-2}$ . It falls well within the observed forcing between  $1.86 \text{ Wm}^{-2}$   
526 and  $3.33 \text{ Wm}^{-2}$ . The heat of deforestation contributed nearly 36% of the total heat added to the  
527 surface, assuming energy production between 1850 and 1990 to be linear with time. Clearly, the  
528 heat of deforestation is large and substantial. The calculated maximum and average values of  
529 seasonal efficiency of photosynthesis are 0.013 and 0.007 respectively, Section 6. Jørgensen and  
530 Svirezhev (2004) compiled average values of photosynthesis efficiency available in the  
531 literature. The green leaf machine model yielded 0.016 and the radiative models yielded values

532 between 0.005 and 0.007. The calculated seasonal efficiency of photosynthesis is in agreement  
533 with the work of others. Table 1, figures 3 and 4, and the discussion section show good  
534 agreement between calculated and observed world populations. At the present decreasing trend  
535 of deforestation, world population projections are 8.4 billion in 2030; 9.4 billion in 2050; and  
536 practically steady at 10.3 billion in 2100. For comparison, UN (2019) projections are 8.5 billion  
537 in 2030; 9.7 billion in 2050; and steady at 10.9 billion in 2100. Should deforestation remain at its  
538 current value of 0.08% annually, the world population is expected to reach 14.5 billion instead.  
539 Based on these agreements between calculations and observed heat added to the surface,  
540 population size, and photosynthesis efficiency, equations (11) and (20) may be used to calculate  
541 the heat of deforestation and the size of world population. The only variable required for these  
542 equations is the average annual deforestation fraction.

543 The population function, Eq. (20), is derived on the basis that population size is proportional  
544 to the size of the farmland available that provides carrying capacity of the biosphere. The  
545 equation is in essence a logistic growth applicable to organisms and plants as well. The carrying  
546 capacity,  $N$  of Eq. (17), would be available after land has been cleared and converted into  
547 productive farmland. Population then grows exponentially, Eq. (18), until the carrying capacity  
548 of the biosphere has been satisfied, and the process repeats again with the addition of new  
549 farmland. Apparently, there is a similarity between human population growth and growth of  
550 other species. The difference is that the cleared land does not provide readily digestible nutrients.  
551 It takes years for the cleared land to be converted into productive farmland and increase land  
552 carrying capacity. The logistics associated with this process appear to be embedded in human  
553 population growth as discussed in the discussion section. Population size lags new land carrying  
554 capacity by nearly ten years. It is thus reasonable to conclude that world population growth is a  
555 dependent parameter; it depends on the cleared and farmed land. So is energy production  
556 because it is a function of population size. Therefore, deforestation and farming of the cleared  
557 land may be considered as regulators of population size and climate.

558 The errors in the calculated heat of deforestation and population size variation by equations  
559 (11) and (20) are nearly equal to the errors in deforestation measurements. The data sources do  
560 not provide error margins of average annual deforestation fractions. As a result, calculation  
561 errors are not presented. Lack of data relative to energy consumption prior to 1990 necessitated  
562 the assumption of linearity of energy consumption with time. In reality the energy consumption  
563 curve lies below the assumed line. This is evident from the world population trend before 1990  
564 (Cain et al. 2014, UN 2019) and population Eq. (20). Therefore, the contribution of deforestation  
565 to surface warming is greater than the calculated 36%, and it may well approach 50%.

566 Finally, there is room for improvement and research relative to the fundamental three  
567 variables required for equations (11) and (20). These are seasonal efficiency of photosynthesis,  
568  $\eta$ ; duration of the life cycle,  $n$ ; and average annual deforestation fraction,  $d$ . Climate change is  
569 presently increasing surface temperature and the content of carbon dioxide in the atmosphere.  
570 Therefore, seasonal efficiency of photosynthesis is incrementally increasing and this increase  
571 should be accounted for in the calculations, given the exponential nature of the equations. Also,  
572 duration of the life cycle may be variable with technological advancement and may assume  
573 values other than ten years in the future. The reason is that communication and information  
574 transfer have become practically immediate thus affecting business transaction times and  
575 delivery schedule of agricultural products. Moreover, advancement in technology and  
576 improvement in agriculture practices may impact cycle duration as well. Deforestation is a

577 variable required for calculating population size and heat of deforestation. Consequently,  
578 determining deforestation values with accuracy is important for representative projection of these  
579 climate parameters.

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667 **11. Tables**

668  
669 **Table 1.** The observed and calculated world populations. The observed populations are obtained  
670 from UN (2019).

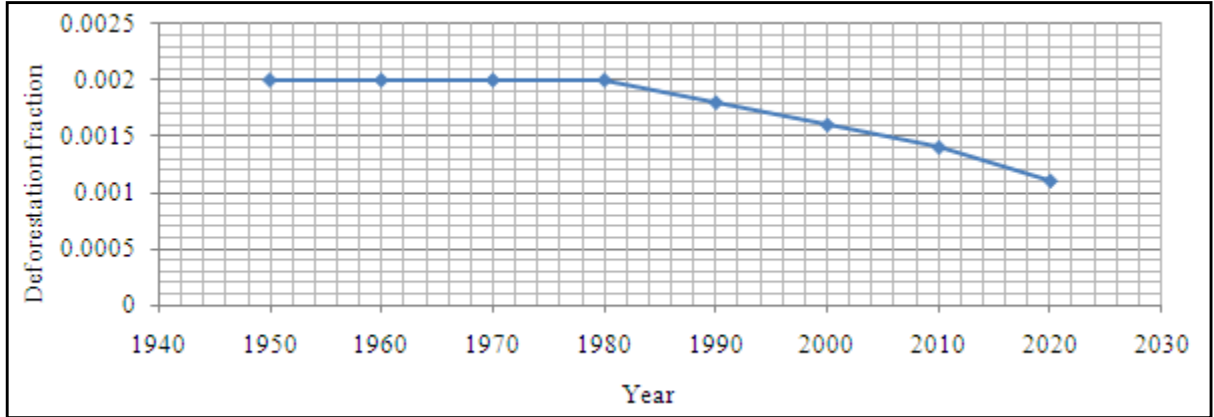
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Year	Average deforestation fraction	Observed population	Calculated population	Difference Percent
1950	0.0020	2.536E+09	2.536E+09	0.00
1960	0.0020	3.035E+09	3.057E+09	0.72
1970	0.0020	3.700E+09	3.684E+09	-0.42
1980	0.0020	4.458E+09	4.441E+09	-0.38
1990	0.0018	5.327E+09	5.260E+09	-1.27
2000	0.0016	6.144E+09	6.119E+09	-0.41
2010	0.0014	6.957E+09	6.991E+09	0.49
2020	0.0011	7.795E+09	7.771E+09	-0.30

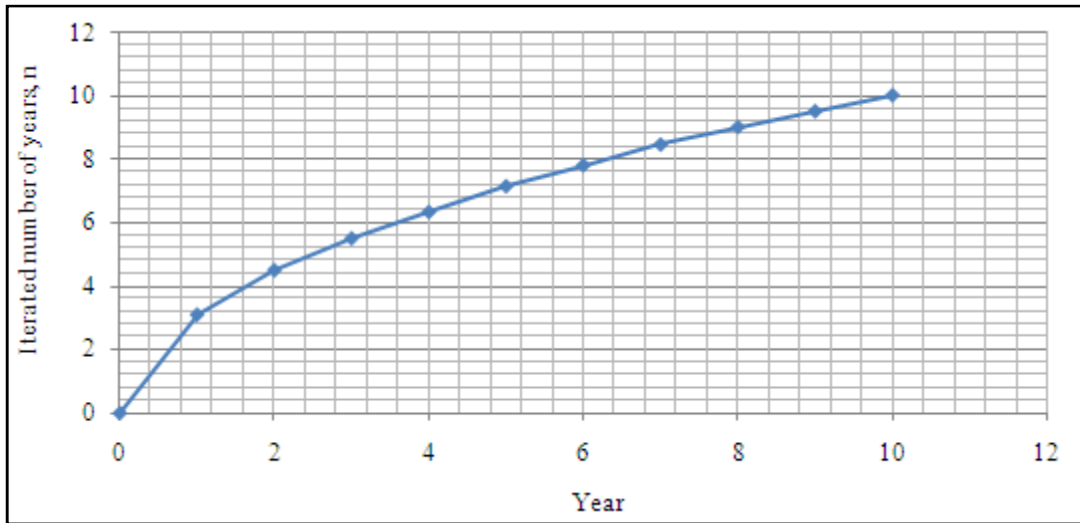
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712 **12. Figures**

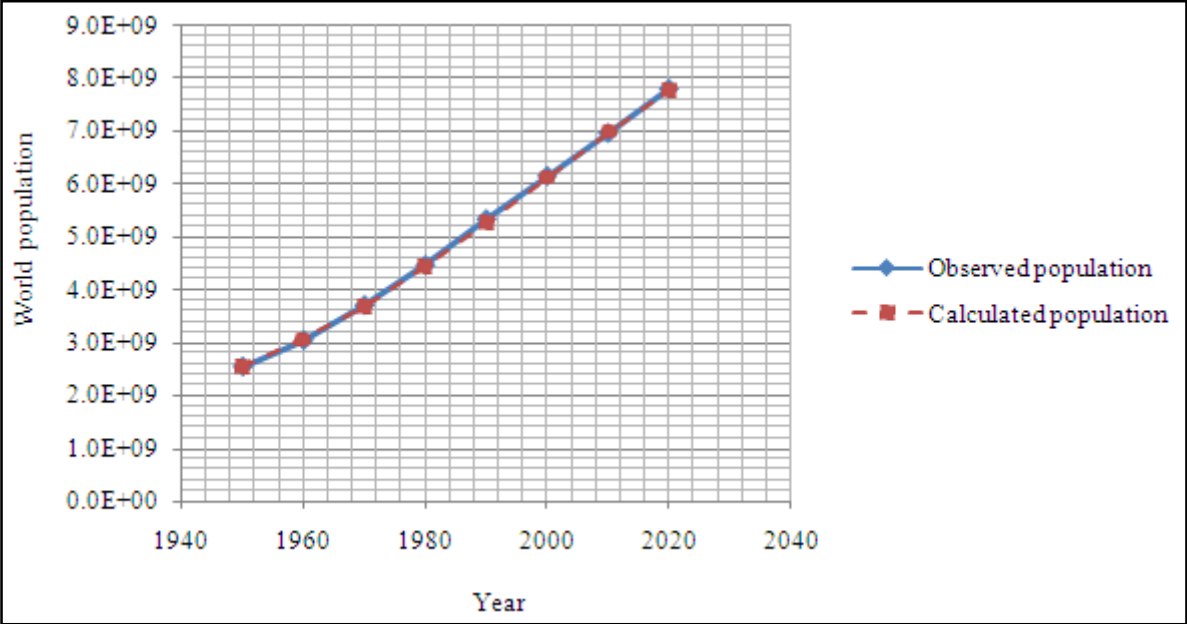


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732 **Fig. 1.** The observed average annual deforestation fractions used in the calculations, obtained  
733 from FAO (2016b).



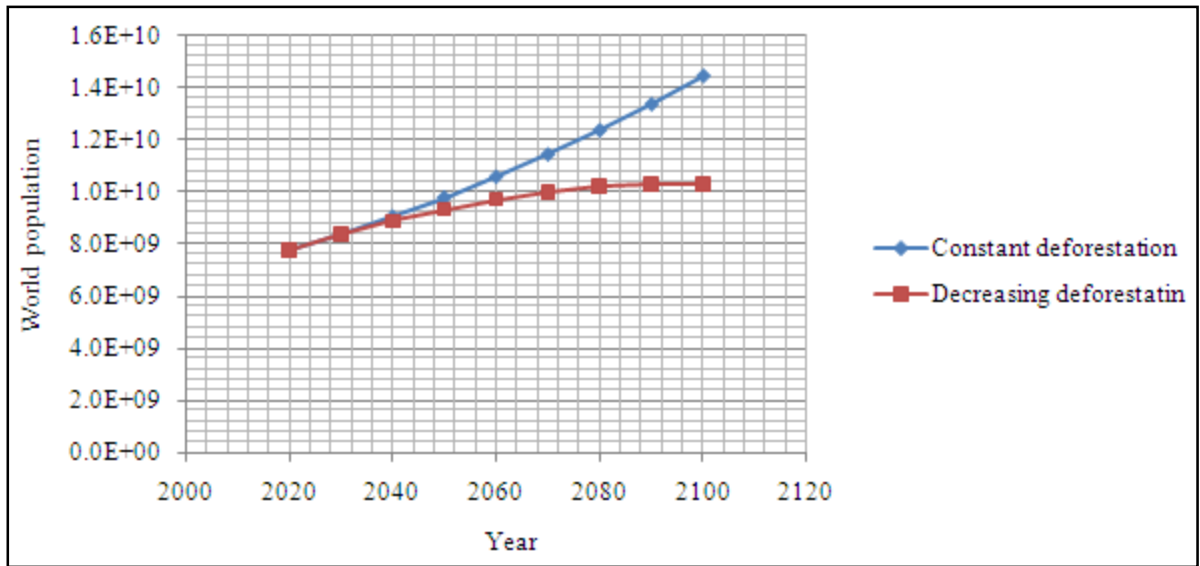
755 **Fig. 2.** Values of the iterated number of years,  $n$ , at which the calculated populations by Eq. (20)  
756 converge to the observed populations. The sample used is for world populations between 1970  
757 and 1980. The population of 1970 is assumed to be a reference population,  $P_0$ , when year=0.  
758 Only for,  $n$ , nearly equal to ten do the years assume the same value. This is typical for any  
759 human population sample.

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**Fig. 3.** Observed and calculated world populations, produced from Table 1.

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**Fig. 4.** World population projections at constant deforestation rate of 0.08% annually and decreasing deforestation. The decreasing trend is assumed to be linear with time, from its present value of 0.08 % annually to negligible in 2100.

868 **13. Symbols and Abbreviations**

869

870

871  $d$  Average annual deforestation fraction, equal to the ratio between

872 the size of the green matter cleared annually and the size of the

873 available green matter, dimensionless.

874  $dn_G'$  Average change in the number of moles of the green matter in a

875 period of time, mol

876  $dn_G$  Maximum change in the number of moles of the green matter in a

877 period of time, mol

878  $dn_D$  Maximum number of moles of the green matter cleared annually,

879 mol

880  $n_{Gnet}$  Net increase in the number of moles of the green matter after

881 subtracting natural biomass decay, mol

882  $dppmv_{CO_2}$  Variation in the concentration of carbon dioxide in the atmosphere

883 in parts per million by volume, ppmv

884  $e$  Natural logarithm base number, equals to 2.7183

885  $F$  Size of the farmland available in the biosphere,  $m^2$

886  $mol$  Molar concentration

887  $\eta$  Average value of the efficiency of seasonal photosynthesis, equal

888 to the ratio between variation in the size of the green matter and

889 total size of the green matter in the biosphere, dimensionless

890  $\eta_{max}$  Maximum value assumed by the efficiency of photosynthesis in a

891 complete seasonal cycle, dimensionless

892  $N$  The maximum population size an ecosystem can sustain, number

893 of species

894  $N_0$  The maximum population size of an ecosystem at a given

895 reference time, number of species

896  $n$  Number of years, years

897  $n_G$  Number of moles of the green matter in the biosphere, mol

898  $ppmv_{CO_2}$  Number of moles of carbon dioxide in the atmosphere in parts per

899 million by volume, ppmv

900  $P_0$  Population size at reference year  $n_0$ , number of species

901  $P(n)$  Population size at an arbitrary year  $n$ , number of species

902  $ppmv$  Concentration of carbon dioxide in the atmosphere in parts per

903 million by volume

904  $Q_G$  The chemical energy stored in the green matter, Joule

905  $Q_D$  The heat of decay, combustion, or oxidization resulting from

906 clearing green matter, Joule

907  $r$  Population exponential growth rate,  $t^{-1}$

908  $r_F$  Variation in the size of farmland with time,  $m^2 t^{-1}$

909  $t$  Age of the green matter or time, years

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